Review of some Concepts from High School

Here are a few concepts from high school mathematics that are very useful in calculus.

Adding Fractions

To add two fractions a/b and c/d, the denominator needs to be the same. To do this, you find a common denominator for the two fractions. For example, to add 2/3 and 4/5, you first give them the common denominator $3 * 5 = 15$. So 2/3 becomes 10/15, and 4/5 becomes 12/15. You can then add the numerators of the fractions:

$$\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15}$$

Here is an example with variables:

$$\frac{x+1}{x+2} + \frac{y}{x} = \left(\frac{x}{x}\right)\left(\frac{x+1}{x+2}\right) + \left(\frac{x+2}{x+2}\right)\left(\frac{y}{x}\right)$$

them simplify:

$$= \frac{x(x+1)}{x(x+2)} + \frac{y(x+2)}{x(x+2)} = \frac{x(x+1)+y(x+2)}{x(x+2)}$$

Dividing Fractions

In general, to divide one fraction a/b by another c/d, one uses the "invert and multiply" rule:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} * \frac{d}{c}$$

For example,

$$\frac{\frac{2}{3}}{\frac{5}{6}} = \frac{2}{3} * \frac{6}{5} = \frac{2}{1} * \frac{2}{5} = \frac{4}{5}$$

Exponentian Rules

When multiplying exponents with the same base, one adds the exponents. For example,

$$2^3 * 2^5 = 2^{3+5} = 2^8$$

or

$$x^6 * x^3 = x^{6+3} = x^9$$

When taking an exponent to a power, one multiplies the exponents. For example,

$$(2^3)^5 = 2^{3*5} = 2^{15}$$

or

$$(x^6)^3 = x^{6*3} = x^{18}$$

Rationalizing

It is often helpful, if one has an expression with a root on the bottom, to move the root to the top by rationalizing. To do this, one multiplies the top and bottom parts by the conjugate of the denominator. For example, to rationalize

$$\frac{42}{1-\sqrt{7}}$$

you multiply top and bottom by $1+\sqrt{7}$. So rationalizing would look like this:

$$\frac{42}{1-\sqrt{7}} = \left(\frac{42}{1-\sqrt{7}}\right)\left(\frac{1+\sqrt{7}}{1+\sqrt{7}}\right)$$

then simplify:

$$= \frac{42(1+\sqrt{7})}{1-\sqrt{7}+\sqrt{7}-\sqrt{7}\sqrt{7}} = \frac{42(1+\sqrt{7})}{1-7} = \frac{42(1+\sqrt{7})}{-6} = -7(1+\sqrt{7})$$

The Slope of a Line

If you know two points on a line (x_1, y_1) and (x_2, y_2), then the slope is given by the formula

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

For example, the slope of the line with points $(5, 2)$ and $(3, 1)$ is

$$\text{slope} = \frac{1-2}{3-5} = \frac{-1}{-2} = \frac{1}{2}$$

Finding the Equation of a line

If you have the slope (call it m) of the line and a point (x_1, y_1) on the line, tthen the equation of the line is:

$$y = m(x - x_1) + y_1$$

For example, if we have the slope $2/3$ and the point $(4, 5)$, then the equation of the line is

$$y = 2/3(x - 4) + 5 = 2/3x - 8/3 + 5 = 2/3x + 7/3$$

The equation should be written in its final form:

$$y = 2/3x + 7/3$$

Functions

A function is a rule that assigns an element of a set to a element of another set. Most of the functions we will deal with in calculus assign a real number to another real number. For example, the function $f(x) = x^2$ takes a real number x and returns x^2. Sometimes functions are slightly more complicated and have conditions; for example the absolute value function is:

$$|x| = \begin{cases} x & \text{if } x \geq 0; \\ -x & \text{if } x < 0. \end{cases}$$

So for example, $|3| = 3$ while $|-3| = -(-3) = 3$ as well.

Here is another example of a function with pieces:

$$f(x) = \begin{cases} 3 & \text{if } x = 1; \\ x^3 & \text{if } x \neq 1. \end{cases}$$

For more information on functions, please read section 1.1 in the textbook.